A new simple accurate and robust a posteriori subcell finite volume limiter for the discontinuous Galerkin method

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ABSTRACT

In our talk we present a new robust, accurate and very simple *a posteriori* subcell finite volume limiter technique for the Discontinuous Galerkin (DG) finite element method for nonlinear systems of hyperbolic PDE in multiple space dimensions that works well for *arbitrary* high order of accuracy in space and time and that does *not* destroy the natural *subcell resolution* properties of the DG method. High order time discretization is achieved via a fully-discrete one-step ADER approach that uses a local space-time discontinuous Galerkin predictor method to evolve the data locally in time within each cell.

The new limiting strategy is based on a novel *a posteriori* verification of the validity of a discrete candidate solution against physical and numerical detection criteria. In particular, we employ a relaxed discrete maximum principle, the positivity of the numerical solution and the absence of floating point errors as detection criteria. For those troubled cells that need limiting, our new approach *recomputes* the discrete solution by starting again from a valid solution at the old time level, but using a more robust finite volume scheme on a refined subgrid of $N_s = 2N + 1$ subcells, where N is the polynomial approximation degree of the DG scheme. The new method can be interpreted as an *element-local checkpointing* and *restarting* of the solver, but using a more robust scheme on a finer mesh after the restart.

The performance of the new method is shown on a large set of different hyperbolic PDE systems using uniform and space-time adaptive Cartesian grids (AMR), as well as on unstructured meshes in two and three space dimensions.

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